UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING
Department of Electrical &
Computer Engineering

ECE 204 *Numerical methods*

Approximating the derivative using least-squares best-fitting polynomials

Douglas Wilhelm Harder, LEL, M.Math.
dwharder@uwaterloo.ca
dwharder@gmail.com

Introduction

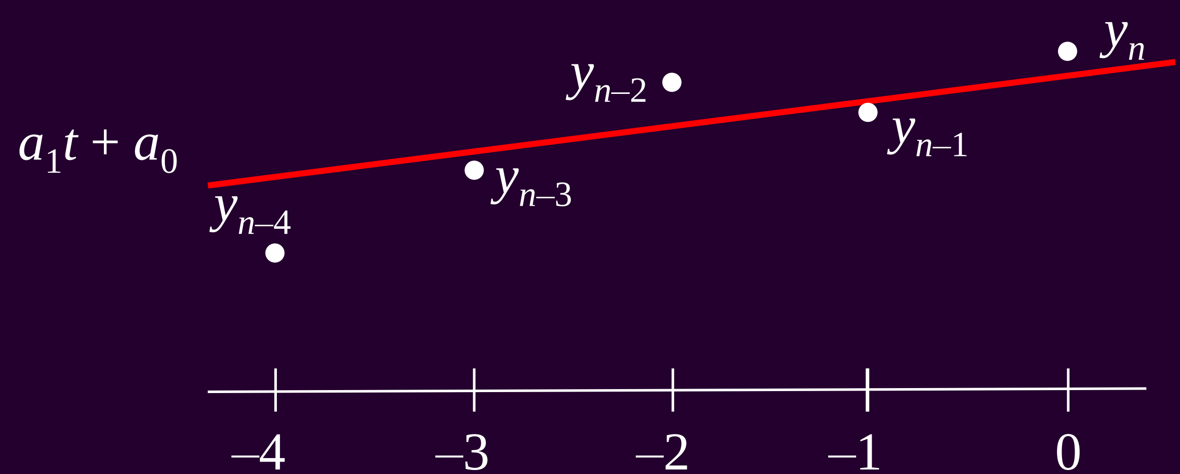
- In this topic, we will
  - Discuss how to estimate the derivative of data by using the least-squares best-fitting polynomials
  - Estimating $y^{(1)}(t_n)$ or $y^{(2)}(t_n)$ where $t_k = t_0 + kh$
  - Describe the formula for both linear and quadratic polynomials
  - For the quadratic polynomial,
    we will also approximate the second derivative

Approximating the derivative

- Suppose we have found the least-squares linear polynomial that passes through N equally-spaced points
  - The slope of the best-fitting linear polynomial we found was

$$a_1 = -0.2y_{n-4} - 0.1y_{n-3} + 0.1y_{n-1} + 0.2y_n$$

  - Issue: this involved scaling, or dividing by h
    - Thus, the best approximation of the derivative is $y^{(1)}(t_n) \approx \frac{a_1}{h}$

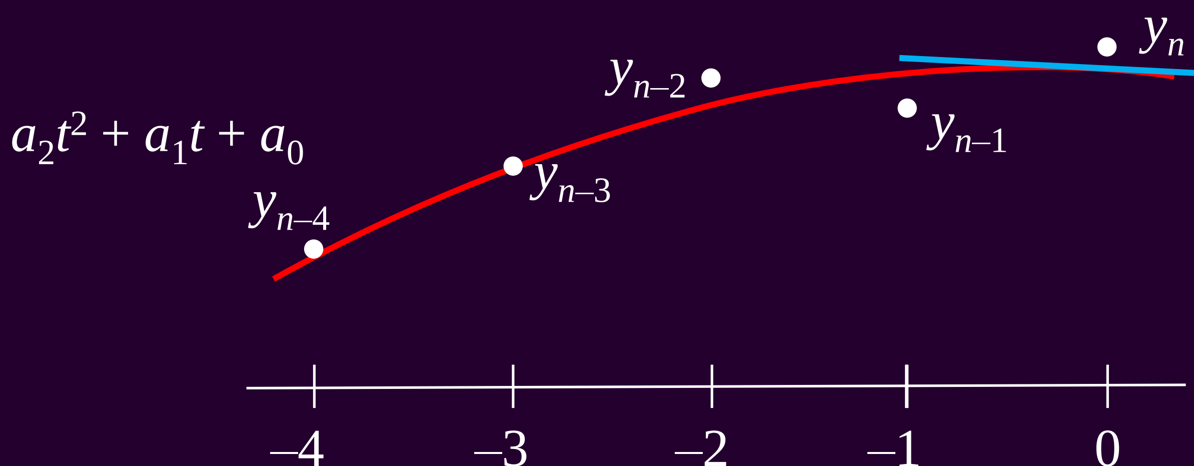


Here, $N = 5$

Approximating the derivative

- Suppose we have found the least-squares quadratic polynomial that passes through N points
  - We found the coefficients of the least-squares quadratic polynomial $a_2t^2 + a_1t + a_0$ which has a derivative $2a_2t + a_1$
  - At time t_n, the best estimate of the derivative is $\frac{a_1}{h}$
  - At time $t_n + \delta h$ the best estimate of the derivative is $y^{(1)}(t_n + \delta h)$

$$\approx \frac{2a_2\delta + a_1}{h}$$



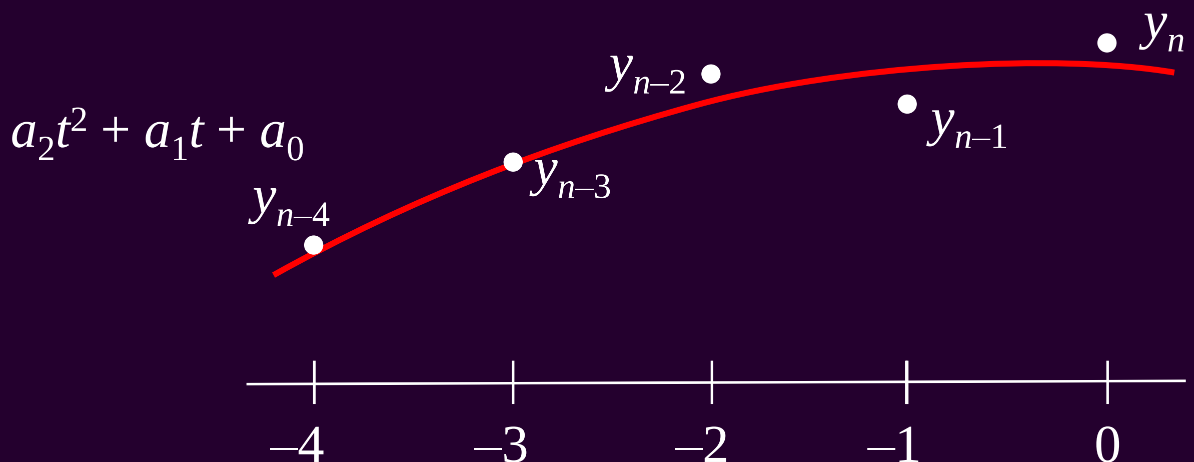
$$\delta \leftarrow \frac{t - t_n}{h}$$

Here, $N = 5$

Approximating the second derivative

- Similarly, we can estimate the concavity with the formula

$$y^{(2)}(t_n) \approx \frac{2a_2}{h^2}$$



Summary

- Following this topic, you now
  - Understand how to estimate the derivative and second derivative of least-squares polynomials
  - Are aware that because our formula involved scaling, we must divide the derivative by the step size, and the second derivative by the step size squared
  - Understand that if we already have the coefficients, we can find these estimates in O(1) time

References

[1] https://en.wikipedia.org/wiki/Least_squares

Acknowledgments

None so far.

Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using `Consolas`. Mathematical equations are prepared in MathType by Design Science, Inc.

Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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for more information.

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